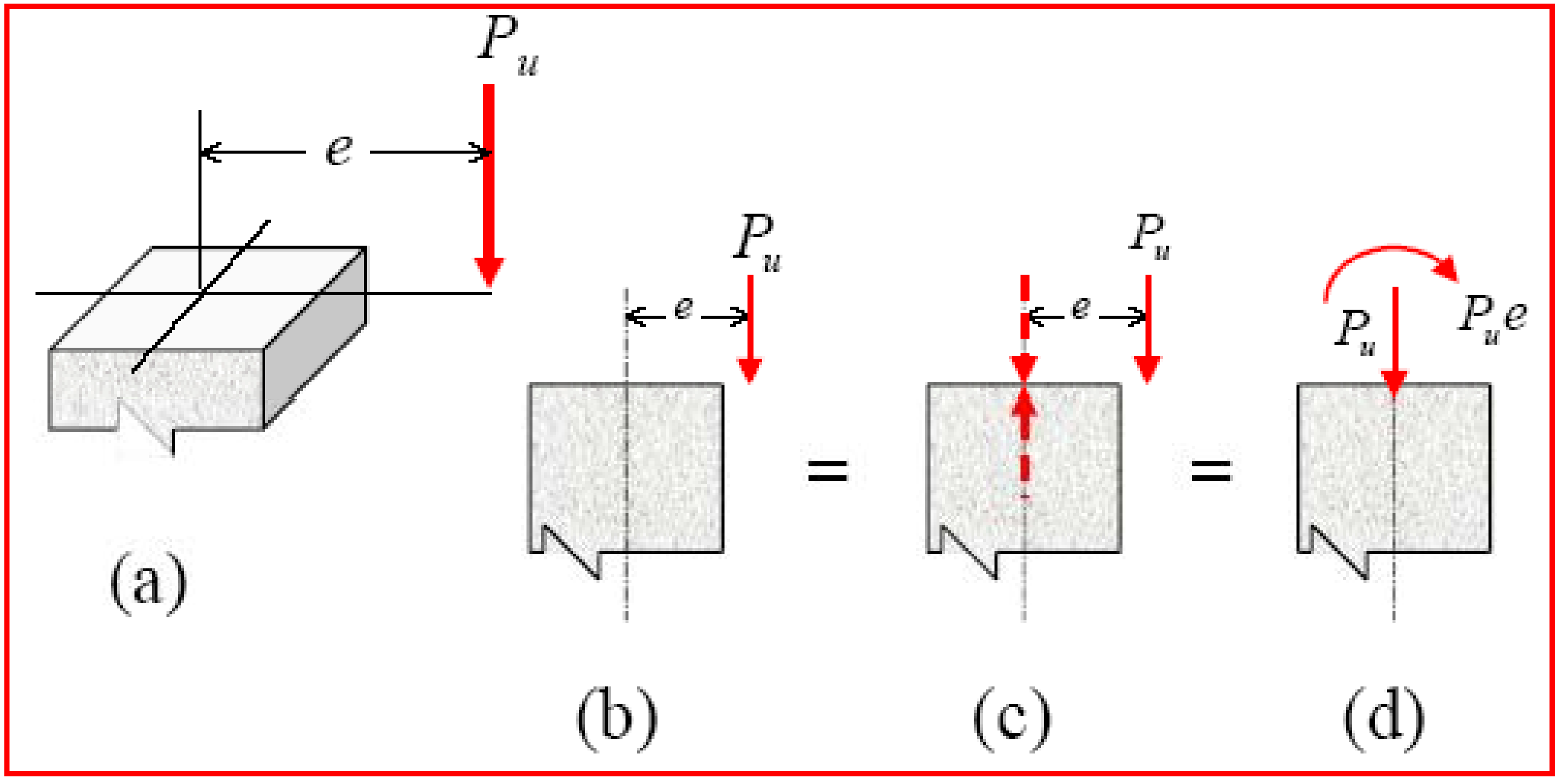
Columns

Review Short Columns (Axial load + Moment)

Usually moment is represented by axial load times eccentricity, i.e.

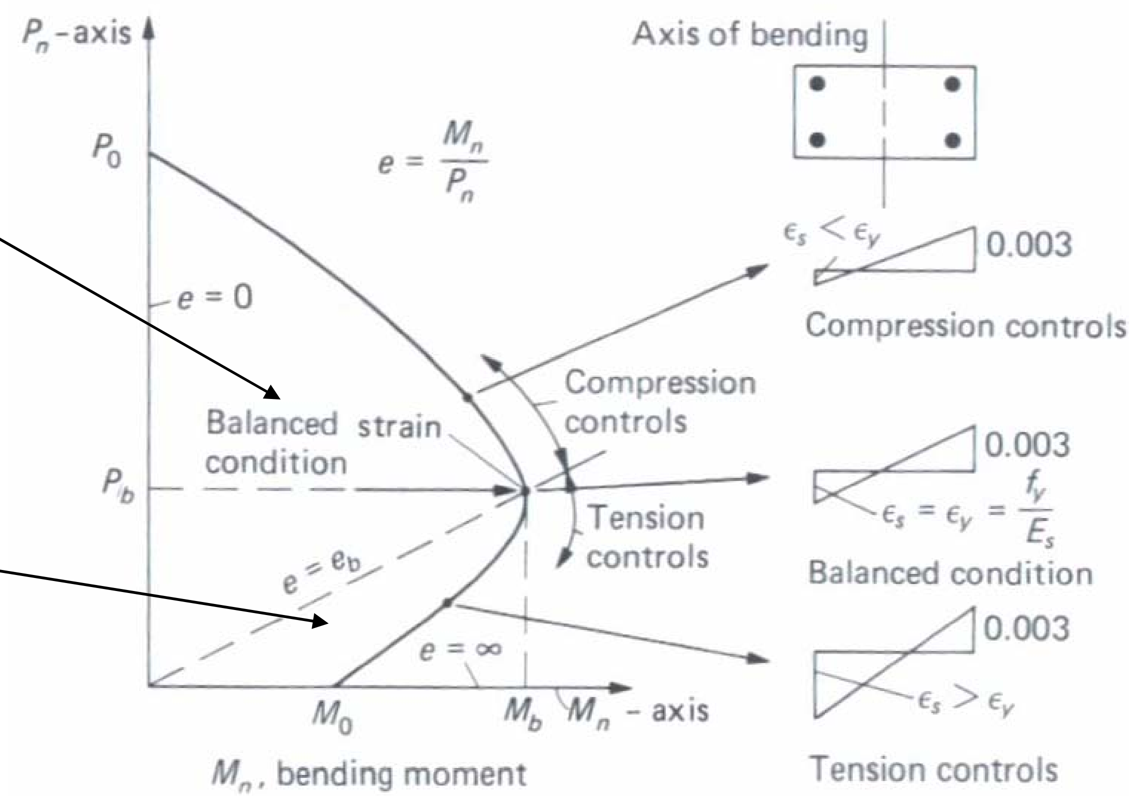


Behavior under Combined Bending and Axial Loads

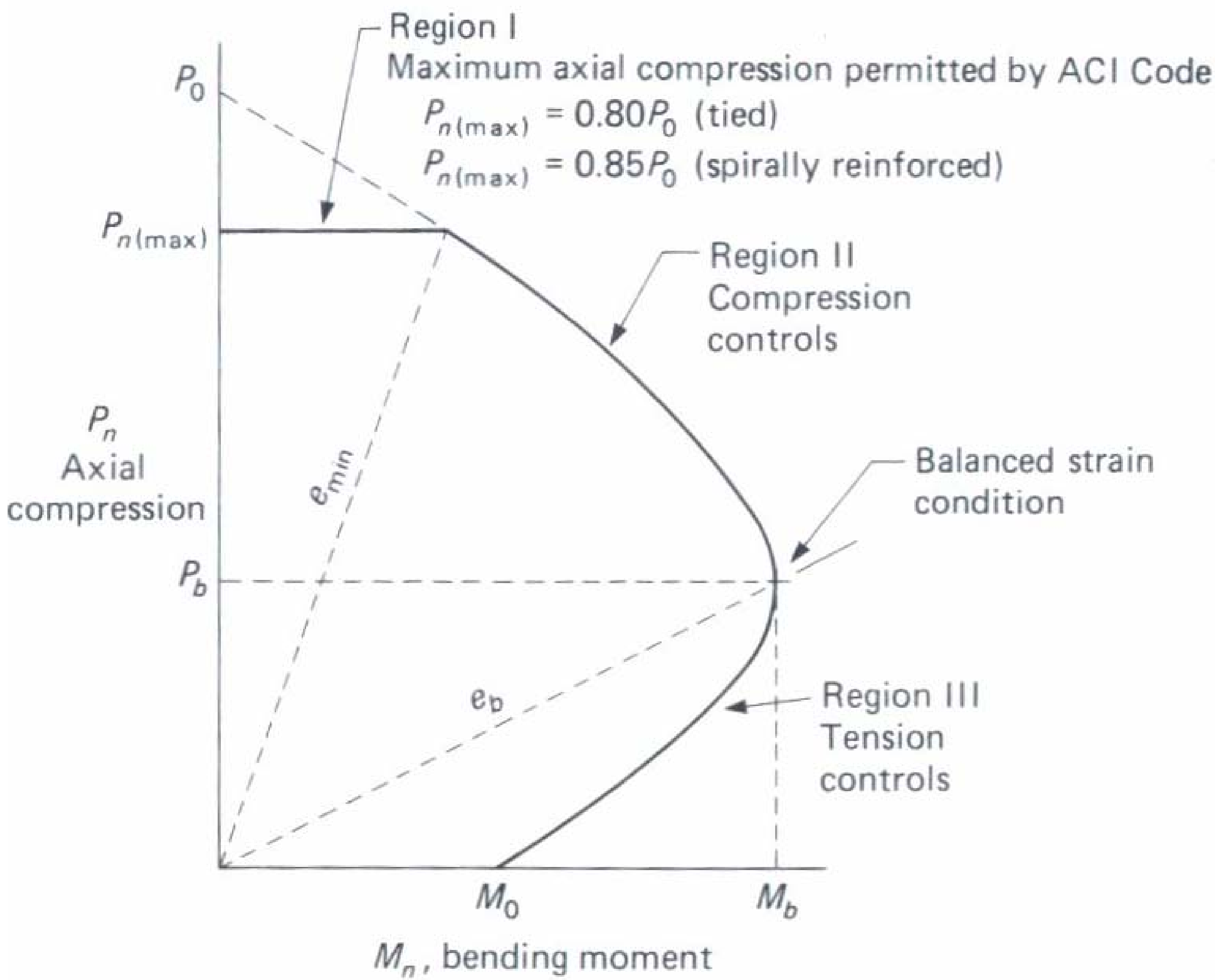
Interaction Diagram Between Axial Load and Moment (Failure Envelope)

Concrete crushes before steel yields

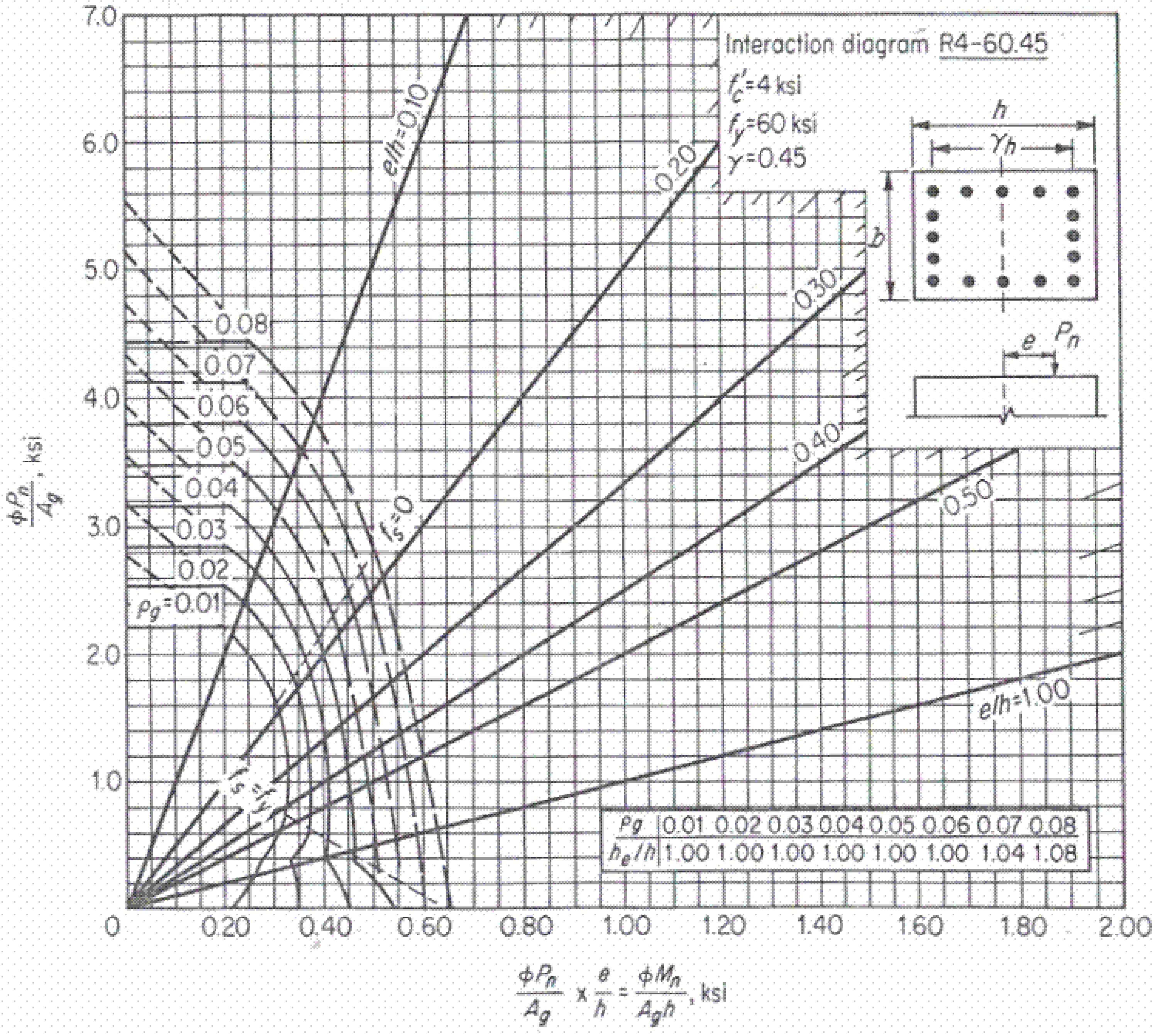
Steel yields before concrete crushes



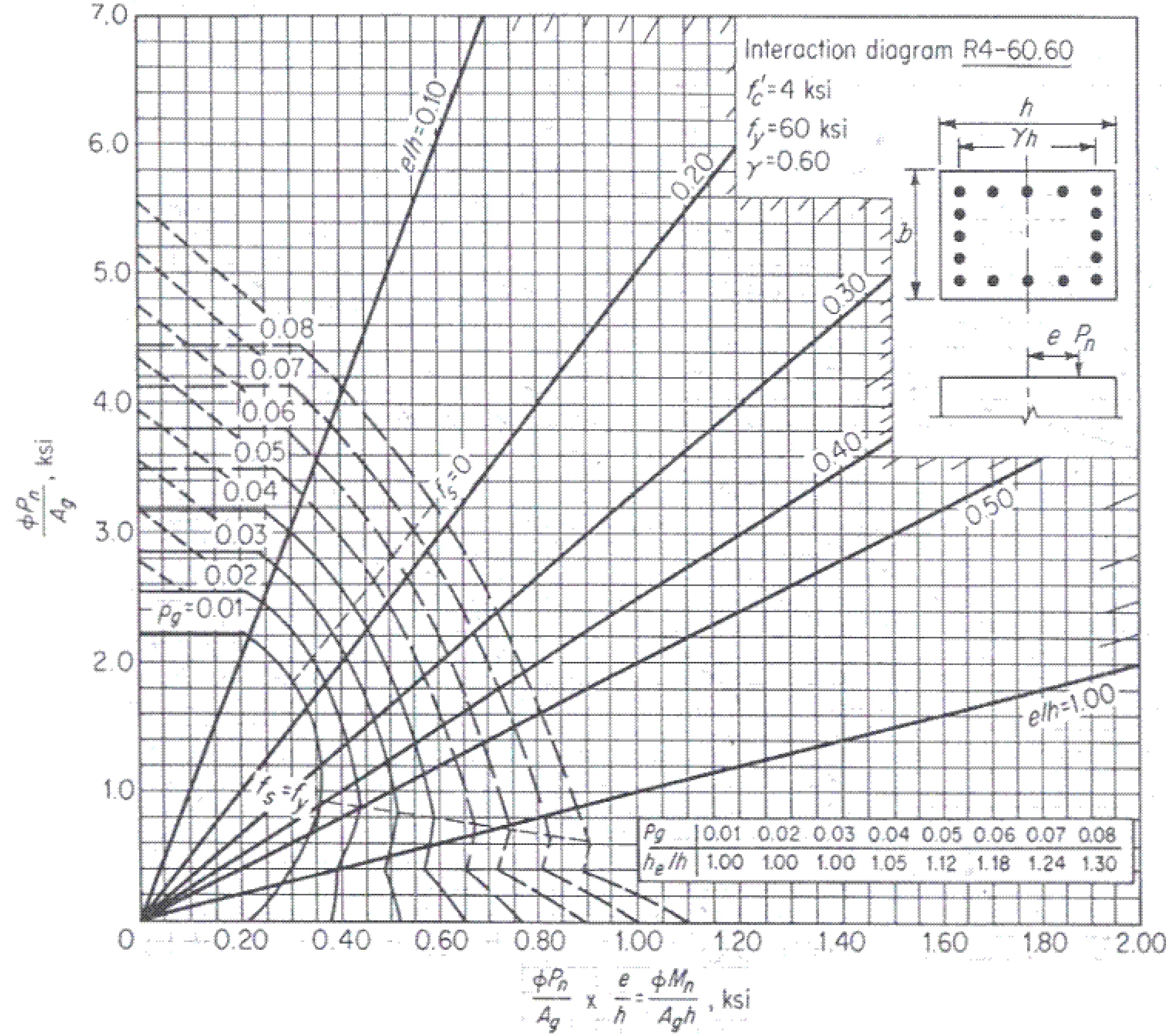
Note: *Any combination of P and M outside the envelope will cause failure.*



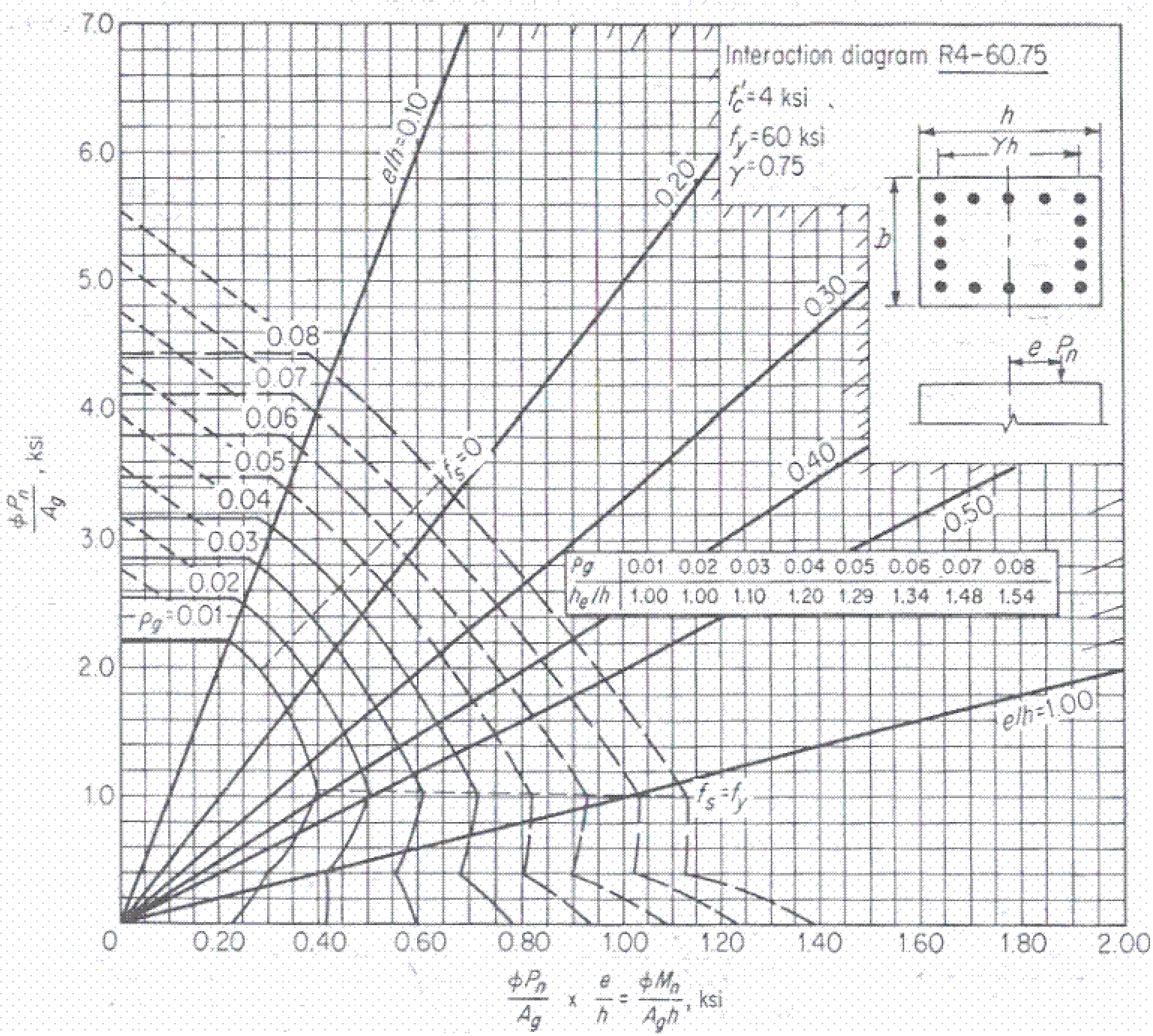
Region I
Maximum axial compression permitted by ACI Code
$P_{n(max)} = 0.80P_0$ (tied)
$P_{n(max)} = 0.85P_0$ (spirally reinforced)
P_0
$P_{n(max)}$
Region II
Compression
controls
P_n
Axial
compression
e_{min}
Balanced strain
condition
P_b
e_b
Region III
Tension
controls
M_0
M_b
M_n, bending moment



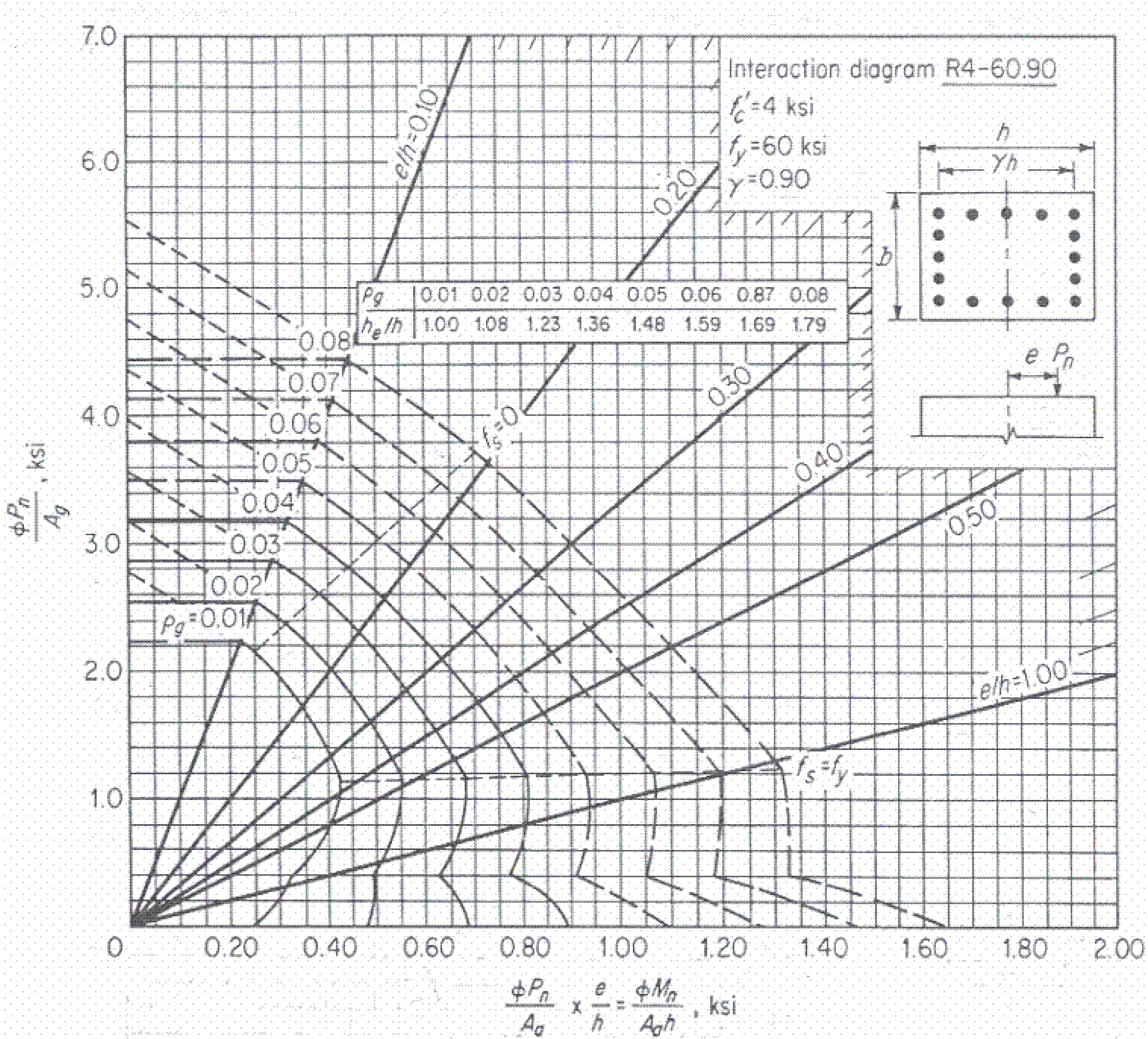
Interaction diagram R4-60.45
$f_c' = 4$ ksi
$f_y = 60$ ksi
$\gamma = 0.45$
h
γh
b
e
P_n
e/h=0.10
0.20
0.30
0.40
0.50
e/h=1.00
$f_s = 0$
$f_s = f_y$
0.08
0.07
0.06
0.05
0.04
0.03
0.02
$\rho_g = 0.01$
ρ_g | 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08
h_e/h | 1.00 1.00 1.00 1.00 1.00 1.00 1.04 1.08
$\frac{\phi P_n}{A_g}$, ksi
$\frac{\phi P_n}{A_g} \times \frac{e}{h} = \frac{\phi M_n}{A_g h}$, ksi



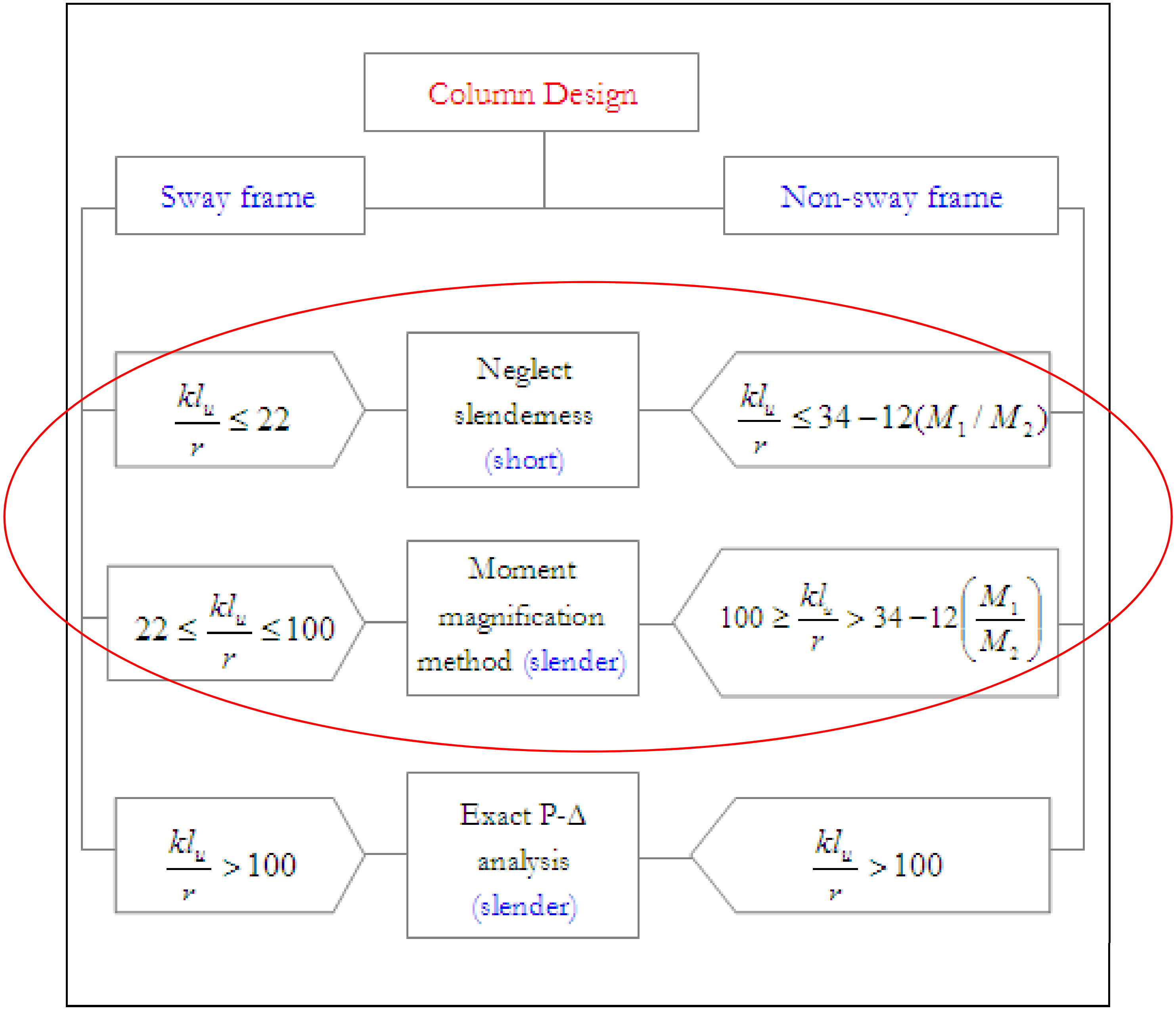
Interaction diagram R4-60.60
f'_c = 4 ksi
f_y = 60 ksi
γ = 0.60
h
γh
b
e P_n
e/h = 0.10
0.20
0.30
0.40
0.50
e/h = 1.00
f_s = 0
$f_s = f_y$
0.08
0.07
0.06
0.05
0.04
0.03
0.02
ρ_g = 0.01
ρ_g | 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08
h_e/h | 1.00 1.00 1.00 1.05 1.12 1.18 1.24 1.30
$\frac{\phi P_n}{A_g}$, ksi
$\frac{\phi P_n}{A_g} \times \frac{e}{h} = \frac{\phi M_n}{A_g h}$, ksi
0 0.20 0.40 0.60 0.80 1.00 1.20 1.40 1.60 1.80 2.00
1.0 2.0 3.0 4.0 5.0 6.0 7.0



Interaction diagram R4-60.75
f'_c=4 ksi
f_y=60 ksi
γ=0.75
h
γh
b
e
P_n
e/h=0.10
0.20
0.30
0.40
0.50
e/h=1.00
f_s=0
$f_s=f_y$
ρ_g=0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
ρg 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08
h_e/h 1.00 1.00 1.10 1.20 1.29 1.34 1.48 1.54
$\frac{\phi P_n}{A_g}$, ksi
$\frac{\phi P_n}{A_g} \times \frac{e}{h} = \frac{\phi M_n}{A_g h}$, ksi



Interaction diagram R4-60.90
f'_c=4 ksi
f_y=60 ksi
γ=0.90
ρ_g 0.01 0.02 0.03 0.04 0.05 0.06 0.87 0.08
h_e/h 1.00 1.08 1.23 1.36 1.48 1.59 1.69 1.79
h
γh
b
e P_n
e/h=0.10
0.20
0.30
0.40
0.50
e/h=1.00
f_s=0
f_s=f_y
0.08
0.07
0.06
0.05
0.04
0.03
0.02
ρ_g=0.01
$\frac{\phi P_n}{A_g}$, ksi
$\frac{\phi P_n}{A_g} \times \frac{e}{h} = \frac{\phi M_n}{A_g h}$, ksi



Column Design
Sway frame
Non-sway frame
$\frac{kl_u}{r} \le 22$
Neglect slenderness (short)
$\frac{kl_u}{r} \le 34 - 12(M_1/M_2)$
$22 \le \frac{kl_u}{r} \le 100$
Moment magnification method (slender)
$100 \ge \frac{kl_u}{r} > 34 - 12\left(\frac{M_1}{M_2}\right)$
$\frac{kl_u}{r} > 100$
Exact P-Δ analysis (slender)
$\frac{kl_u}{r} > 100$

Define Sway & non- sway frame

Stability index

$$Q = \frac{\sum P_u \Delta_o}{V_u l_c}$$

$$Q < 0.05 \Rightarrow \text{Non - } sway \text{(braced)}$$

$$Q > 0.05 \Rightarrow Sway \text{(unbraced)}$$

$\sum P_u$	is the total vertical load in the story
V_u	is the story shear, in the story under consideration
l_c	is length of column measured center-to-center of the joints in the frame, and
Δ_o	is the first-order relative deflection between the top and bottom of that story.

The ACI Procedure for Classifying Short and Slender Column

According to *ACI Code 10.12.2* and *10.13.2*, columns can be classified as short when its effective slenderness ratio satisfies the following criteria:

For non-sway frames $\frac{k\,l_u}{r} \leq 34 - 12\left(M_1 / M_2\right) \leq 40.0$

Or

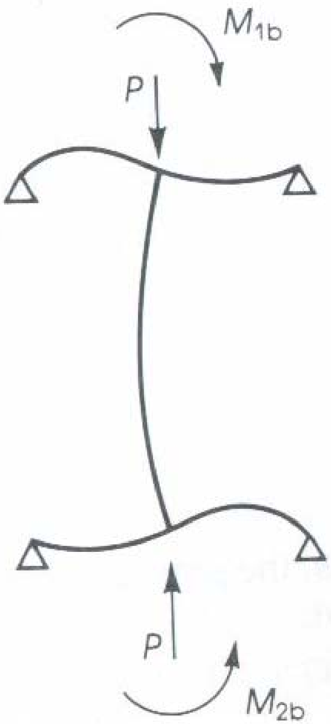
For sway frames $k\,l_u / r \leq 22$

l_u = unsupported length of member, defined in *ACI Code 10.11.3* as clear distance between floor slabs, beams, or other members capable of providing lateral support, as shown

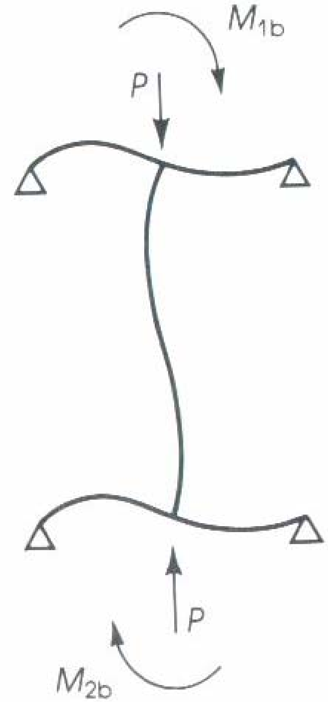
r = radius of gyration associated with axis about which bending is occurring. For rectangular cross sections r = 0.30 h, and for circular sections, r = 0.25 h as specified by ACI Code 10.11.2.

$\mathbf{M_1/M_2 =}$ Ratio of moments at two column ends,

where $M_2 > M_1$ (-1 to 1 range)



$$\frac{M_1}{M_2} > 0$$

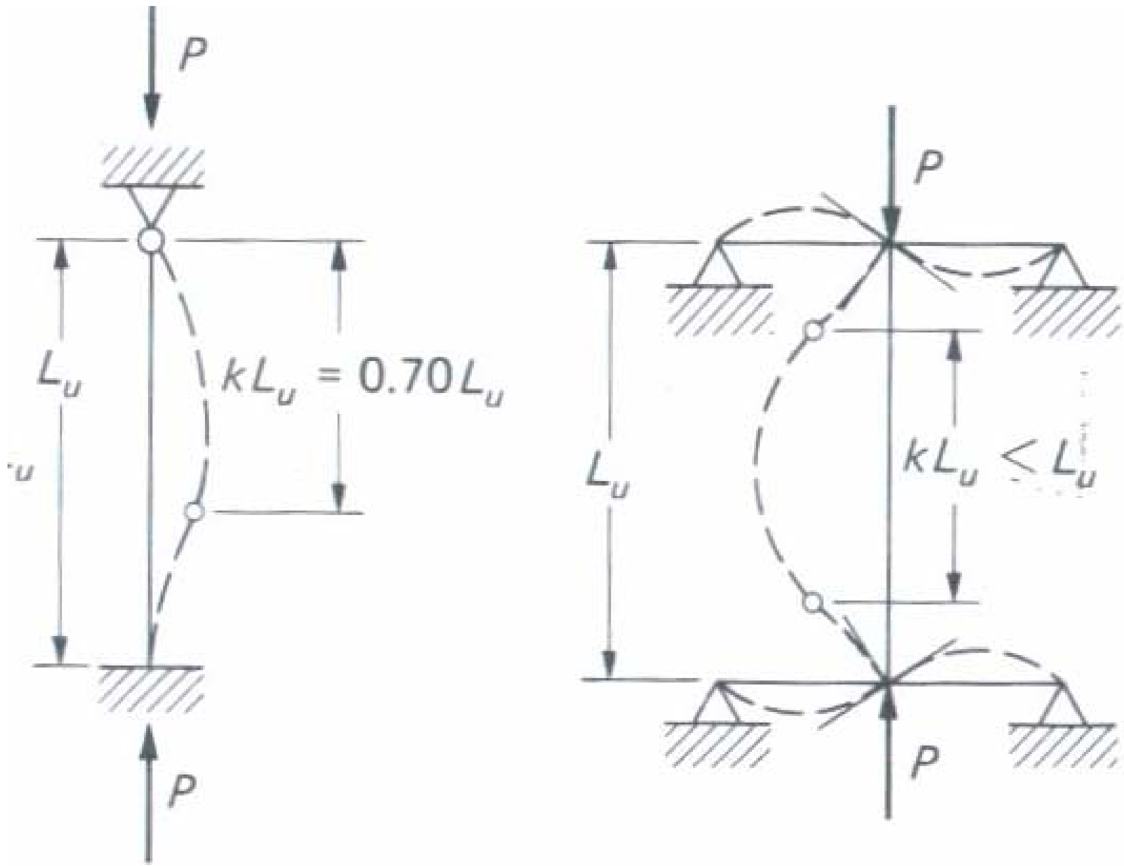


$$\frac{M_1}{M_2} < 0$$

singular curvature

double curvature

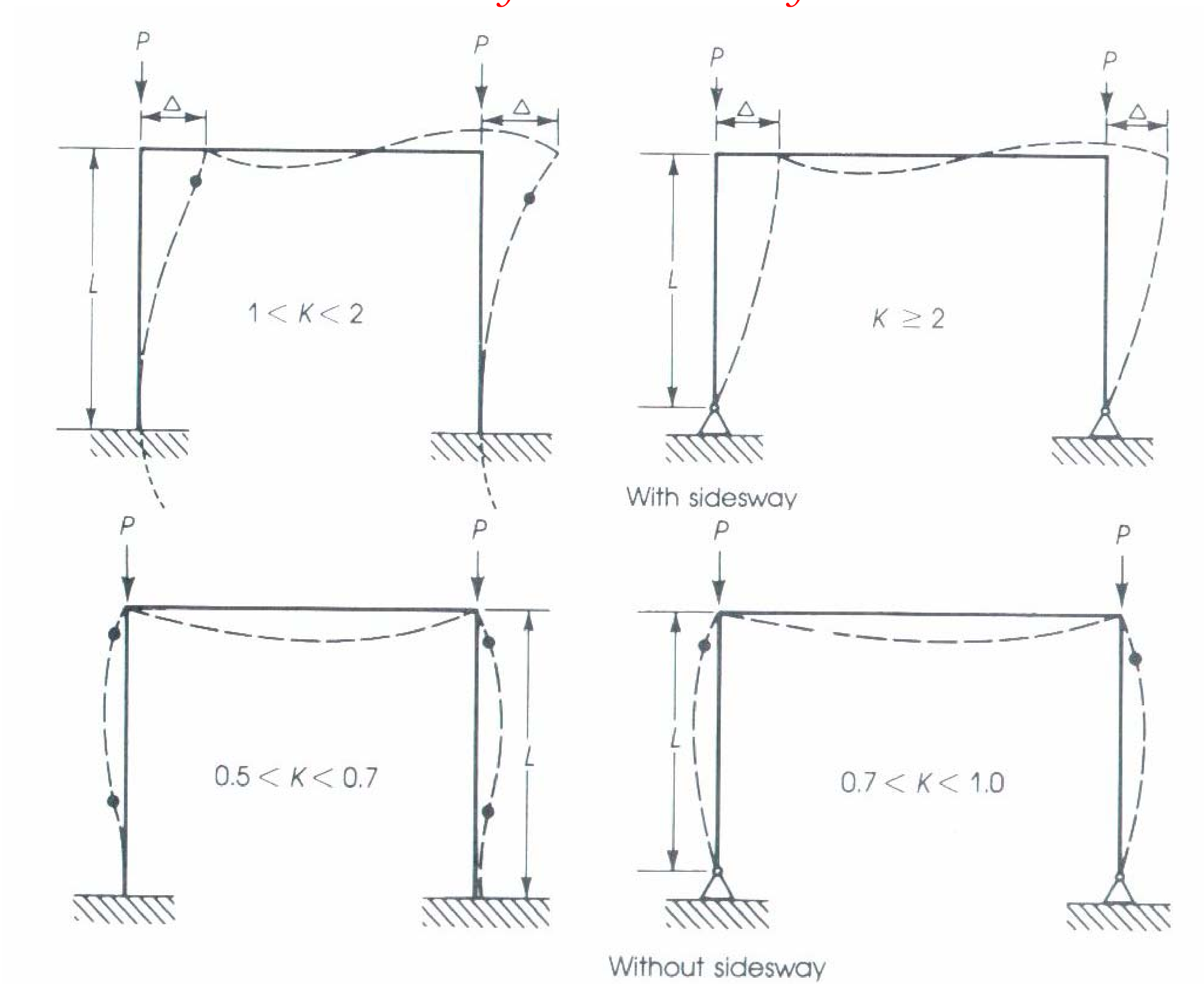
K–Factor = effective length factor



(c) One end restrained, other unrestrained

(d) Partially restrained at each end

Slenderness Ratio for columns in frames



K –Factor calculatoin

$$\psi = \frac{\sum EI / l_c \text{ of columns}}{\sum EI / l_c \text{ of beams}}$$

$I = 0.35 I_g$	for Beam
$I = 0.7 I_g$	for Column
$I = 0.7 I_g$	for Uncracked wall
$I = 0.35 I_g$	for Cracked wall

For a Braced Frame:(Non-sway)

$k =$ smaller of

$$k = 0.7 + 0.05\left(\psi_A + \psi_B\right) \le 1.0$$

$$k = 0.85 + 0.05\,\psi_{\min} \le 1.0$$

For a Sway Frame:

a) Restrained at both ends

if $\Psi_{m} = \Psi_{avg} < 2.0$ $\qquad k = \dfrac{20 - \psi_m}{20} \sqrt{1 + \psi_m}$

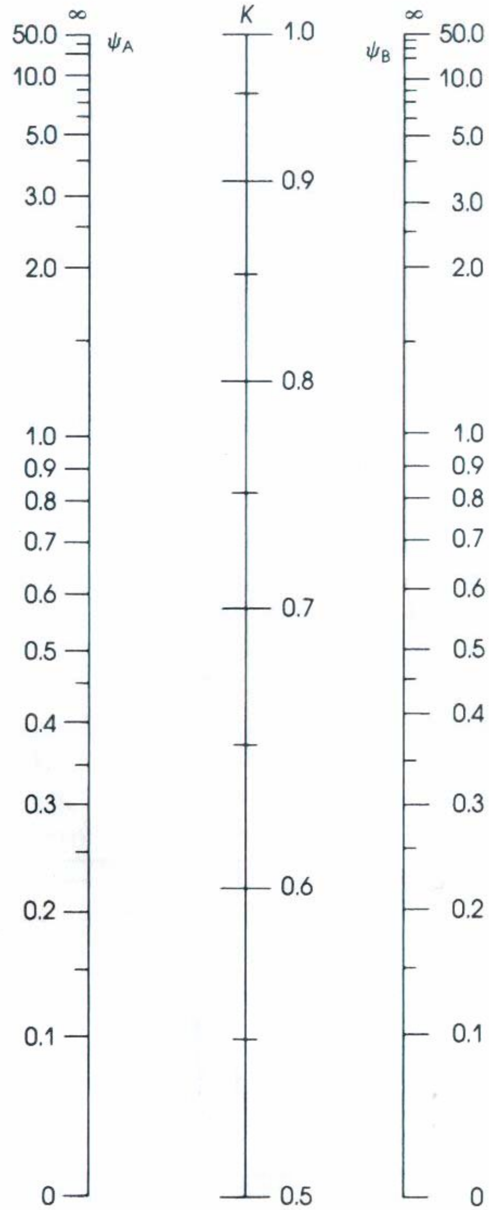
if $\Psi_{m} \geq 2.0$ $\qquad k = 0.9 \sqrt{1 + \psi_m}$

b) One hinged end

$$k = 2.0 + 0.3\,\psi$$

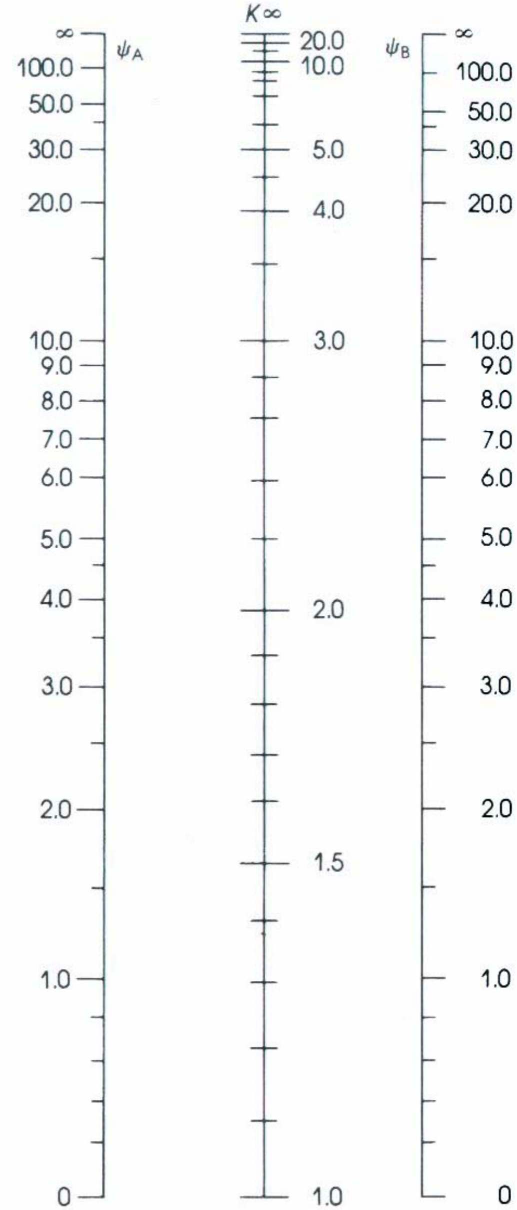
Non-sway frames: $0 \leq k \leq 1.0$

Sway frames: $1.0 \leq k \leq \infty$



$$\psi = \frac{\Sigma\, EI/L \text{ of columns}}{\Sigma\, EI/L \text{ of beams}}$$

Braced frames



$$\psi = \frac{\Sigma\, EI/L \text{ of columns}}{\Sigma\, EI/L \text{ of beams}}$$

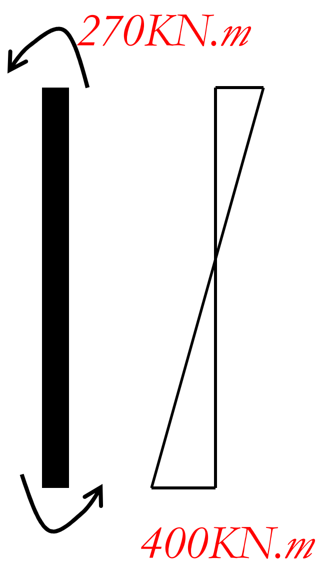
Unbraced frames

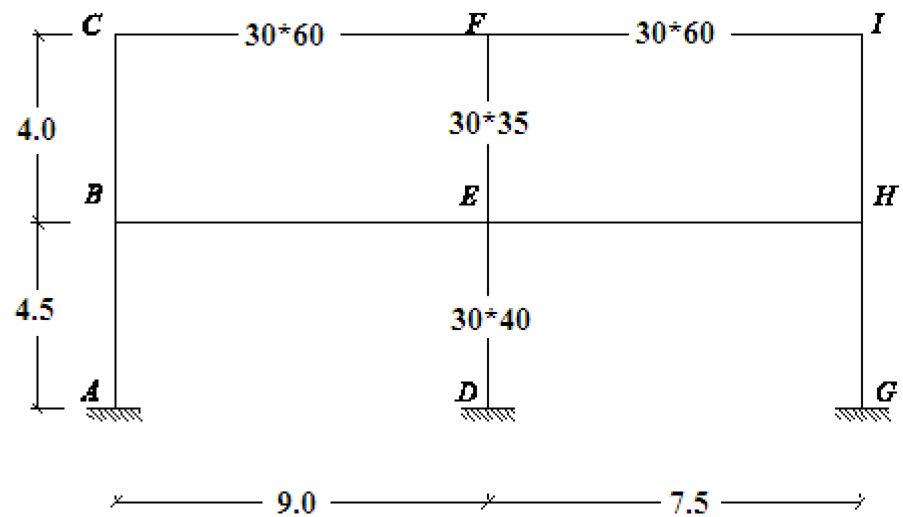
Example 1

The frame shown in Figure is consisting of members with rectangular cross sections, made of the same strength concrete. Considering buckling in the plane of the figure, categorize column ***FE*** as long or short if the frame is:

Nonsway

Sway





Solution:

a. Nonsway:

For a column to be short,

$$\frac{k\,l_u}{r} \le 34 - 12\left(M_1 / M_2\right) \le 40.0$$

$$l_u = 400 - 30 - 30 = 340\ cm = 3400\text{mm}$$

k is conservatively taken as 1.0.

$$k\,l / r = \frac{1(3400)}{0.3(350)} = 32.38$$

$$34 - 12\left(M_1 / M_2\right) = 34 - 12\left(-270 / 400\right) = 42.1 \quad > 32.38$$

Short Column

b. sway:

The column is classified as being short when $k\,l_u / r \le 22$

$$\psi_F = \frac{\left(0.7(30)(35)^3/12(400)\right)}{\left(0.35(30)(60)^3/12(900)\right)+\left(0.35(30)(60)^3/12(750)\right)}$$

$$=0.406$$

$$\psi_E = \frac{\left(0.7(30)(35)^3/12(400)\right)+\left(0.7(30)(40)^3/12(450)\right)}{\left(0.35(30)(60)^3/12(900)\right)+\left(0.35(30)(60)^3/12(750)\right)}$$

$$=0.945$$

Using the appropriate alignment chart, $k = 1.14$, and

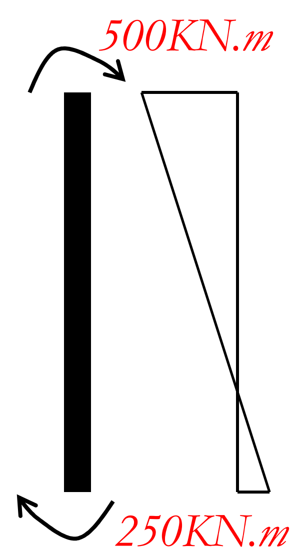
$$\frac{k\,l_u}{r} = \frac{1.14(3400)}{0.3(350)} = 36.91 \quad > 22$$

Column is classified as being slender or long

Example 2

Design reinforcement for a *400 mm x 500 mm* tied column. The column, which is part of a braced frame, has an unsupported length of 3.0 *m*. It is subjected to a factored axial load of 2400 *kN* in addition to a factored bending moment as shown.

$f_c' = 30 Mpa \qquad f_y = 420 Mpa$



Solution:

For this column to be short,

$$\frac{k\, l_u}{r} \leq 34 - 12\left(M_1 / M_2\right) \leq 40.0$$

$$k\, l_u / r = \frac{1(3000)}{0.3(500)} = 20.0$$

$$34 - 12\left(M_1 / M_2\right) = 34 - 12\left(-250/500\right) = 40 \quad > 20.0$$

i.e., the column is classified as being short.

$$\gamma = \frac{50 - 2(4) - 2(1) - 2.8}{50} = 0.744$$

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{2400(1000)}{400(500)} = 12\, N/mm^2 = 12 MPa \; = 1.71 ksi$$

$$\frac{\phi M_n}{A_g\, h} = \frac{M_u}{A_g\, h} = \frac{500(10^6)}{400(500)(500)} = 5 N/mm^2 = 5 MPa \; = 0.71 ksi$$

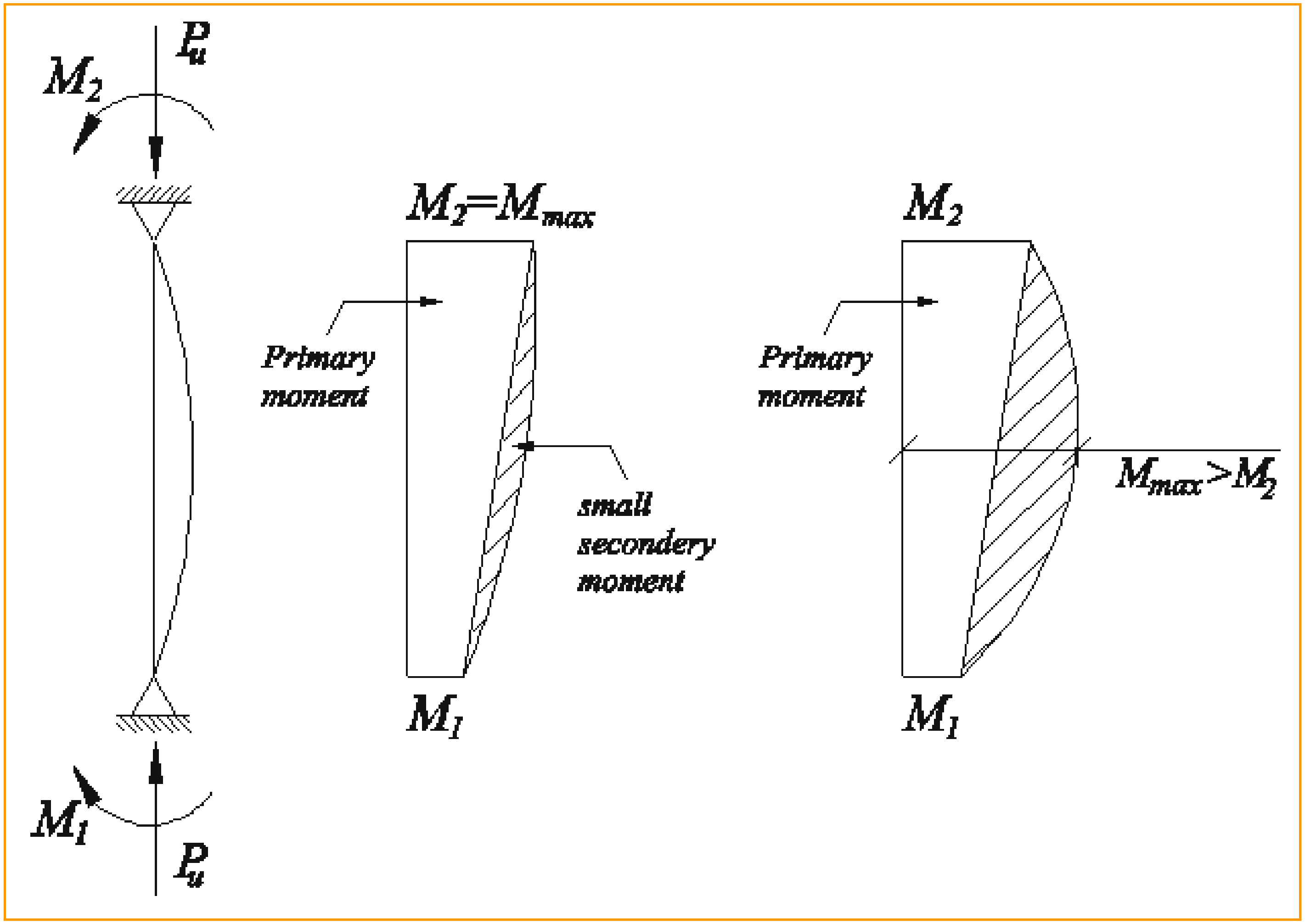
Using the interaction diagram given for $f_c' = 30 Mpa$ $f_y = 420 Mpa$ and $\gamma = 0.75$, one gets $\rho = 0.035$.

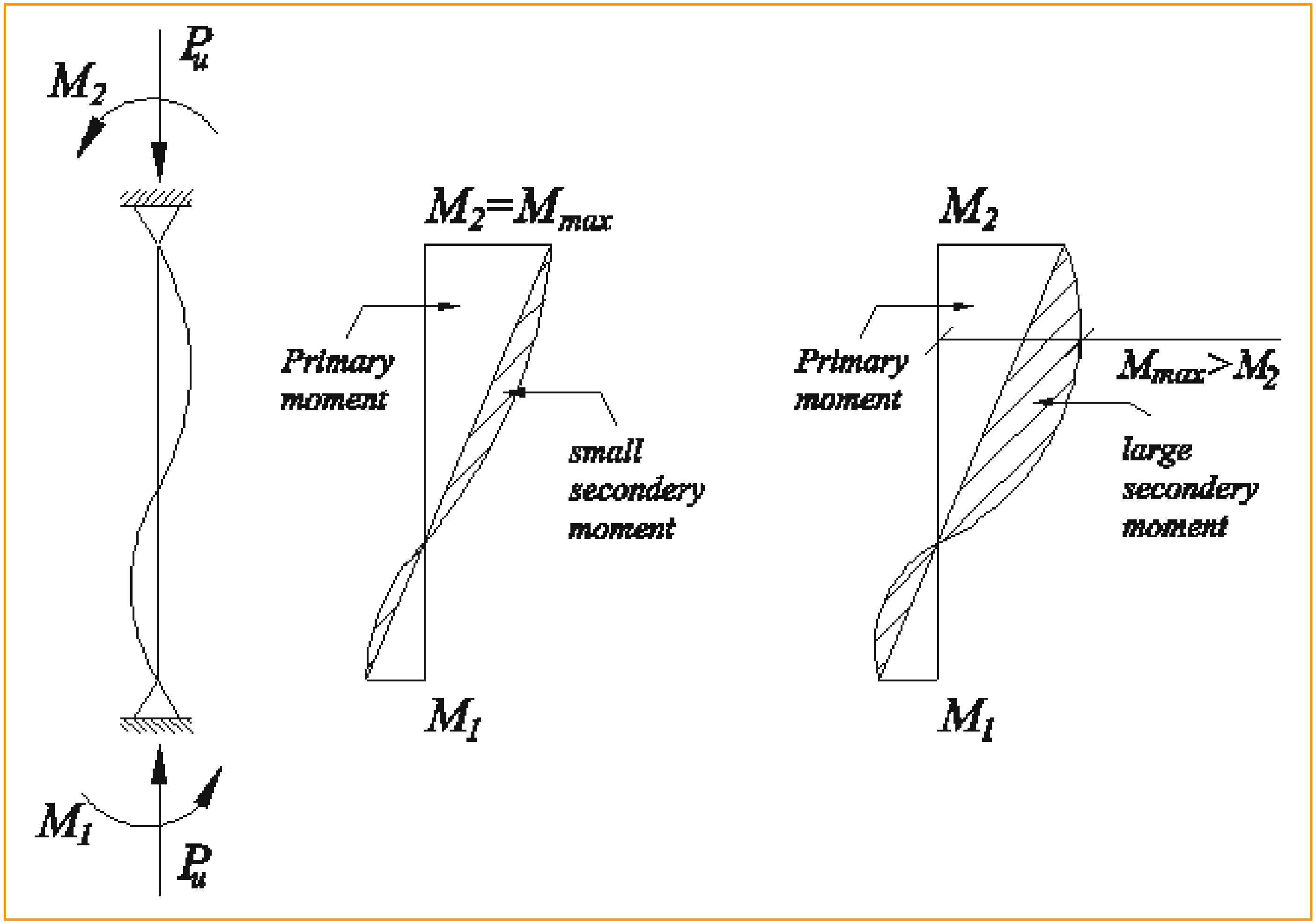
$A_s = 0.035(400)(500) = 7000\, mm^2 = 70 cm^2$, use $14\, \phi\, 25\, mm$

Note to use the last Instruction diagrams (English units) divide $\frac{\phi P_n}{A_g}$ and $\frac{\phi M_n}{A_g\, h}$

By 7.0

Long Columns





P_u
M_2
M_1
P_u
$M_2=M_{max}$
Primary moment
small secondery moment
M_1
M_2
Primary moment
$M_{max}>M_2$
large secondery moment
M_1

Moment Magnification in Non-sway Frames

If the slenderness effects need to be considered. The non-sway magnification factor, δ_{ns}, will cause an increase in the magnitude of the design moment.

$$\boxed{M_{\max} = \delta_{ns} M_2 \quad \geq \delta_{ns} M_{2,\min}}$$

where

$$\boxed{\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75 P_{cr}}} \quad \geq 1.0}$$

Moment Magnification in Non-sway Frames

The components of the equation for an Euler bucking load for pin-end column

$$P_c = \frac{\pi^2 EI}{\left(kl_u\right)^2}$$

and the stiffness, EI is taken as

$$EI = \frac{0.2E_c I_g + E_s I_{se}}{1+\beta_d} \quad \underset{\text{conservatively}}{\Longrightarrow} \quad EI = \frac{0.4E_c I_g}{1+\beta_d}$$

$$\beta_d = \frac{Max.\, factored\ Sustain\ Load}{Max.\, factored\ Axial\ Load}$$

Moment Magnification in Non-sway Frames

A coefficient factor relating the actual moment diagram to the equivalent uniform moment diagram. For members without transverse loads

$$C_{\mathrm{m}} = 0.6 + 0.4\left(\frac{M_1}{M_2}\right) \geq 0.4$$

For other conditions, such as members with transverse loads between supports, $C_m = 1.0$

Moment Magnification in Non-sway Frames

The minimum allowable value of M_2 is

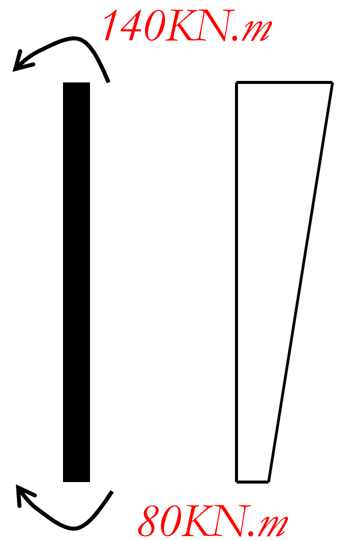
In mm

$$\boxed{M_{2,\min} = P_u\left(15.0 + 0.03\,h\right)}$$

The sway frame uses a similar technique, see the text on the components.

Example 1

Design a 7.0 m-tall column that carries a service dead load of 500 kN, and a service live load of 400 kN, shown in Figure below



Solution:

1- Compute column end moments M_1 and M_2:

$P_u = 1.2(500) + 1.6(400) = 1240\,kN$

$M_1 = 140\,kN.m$

$M_2 = 80\,kN.m$

2- Estimate the column size:

For an assumed reinforcement ratio of 1%, A_g may be assumed as follows:

$$A_g = \frac{P_u}{0.45\left(f_c' + \rho_g f_y\right)} = \frac{1240(1000)}{0.45\left(28 + 0.01(420)\right)} = 85576\,mm^2 = 855.76\,cm^2$$

Try a 50 cm x 50 cm cross section

3- Check whether the column is short or long:

$$\frac{k\,l_u}{r} = \frac{700}{0.3\,(50)} = 46.67 \quad < 100$$

$34 - 12\left(M_1 / M_2\right) = 34 - 12\left(80/140\right) = 27.14$ Long column

4- Evaluate the equivalent moment correction factor C_m:

$$C_m = 0.6 + 0.4\left(M_1 / M_2\right) = 0.6 + 0.4\left(80/140\right) = 0.83 \quad > 0.4 \quad \text{O.K.}$$

5- Evaluate the critical buckling load P_{cr}:

$$\beta_d = \frac{1.2\,(500)}{1240} = 0.48$$

$$E_c = 4775\,\sqrt{f_c'} = 4775\,\sqrt{28} = 25267.0\ N/mm^2$$

$$EI = \frac{0.4\,(25267)(500)(500)^3}{12\,(1+0.48)} = 3.56 \times 10^{13}\ N.mm^2$$

$$P_{cr} = \frac{\pi^2\,(3.56)(10)^{13}}{(7000)^2\,(1000)} = 7170\ kN$$

6- Design the reinforcement:

$$\gamma = \frac{50 - 2(4) - 2(0.8) - 1.6}{50} = 0.776$$

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{1240(1000)}{500(500)} = 5\ N/mm^2 = 0.7ksi$$

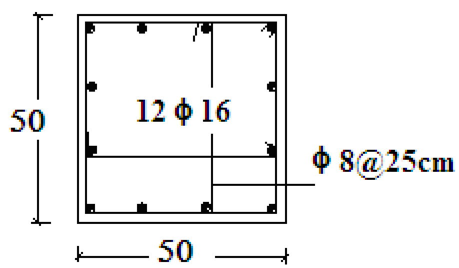
$$\frac{\phi M_n}{A_g h} = \frac{M_u}{A_g h} = \frac{151.2(10^6)}{500(500)(500)} = 1.21\ N/mm^2 = 0.2ksi$$

Using the interaction diagram given for $f_c' = 28 MPa$, $f_y = 420\ MPa$ and $\gamma = 0.75$, one gets $\rho = 0.01$.

$A_s = 0.01(500)(500) = 2500\,mm^2 = 25\,cm^2$, use $12\ \phi 16\,mm$.

Spacing of ties is the smallest of:

- 48 (0.8) = 38.4 cm
- 16 (1.8) = 28.8 cm
- 50 cm



Use three sets of ϕ 8 mm ties @ 25 cm.

Moment Magnification in sway Frames

$$M_{1\max} = M_{1ns} + \delta_s \; M_{1s}$$
$$M_{2\max} = M_{2ns} + \delta_s \; M_{2s}$$

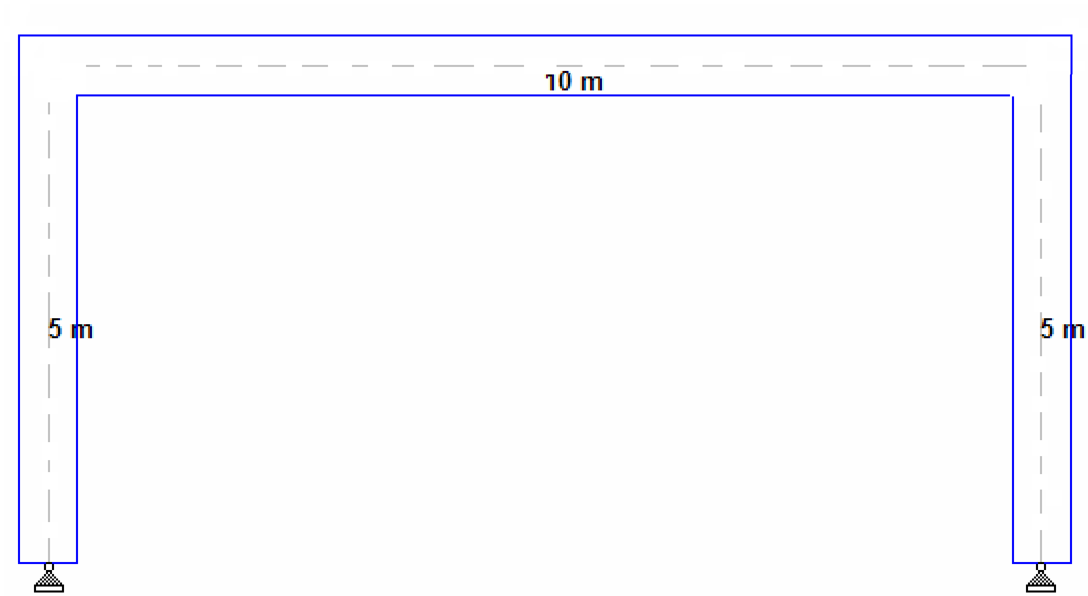
$$\delta_s = \frac{1}{1 - \dfrac{\sum P_u}{0.75 \sum P_{cr}}} \quad \geq \; 1.0 \quad \text{and} < 2.5 \; \text{for } \textit{stability}$$

Example 2

For the frame shown in figure, design column given the following: service dead load including own weight = 60KN/m, service live load = 40KN/m, from left concentrated wind load = 30 KN.

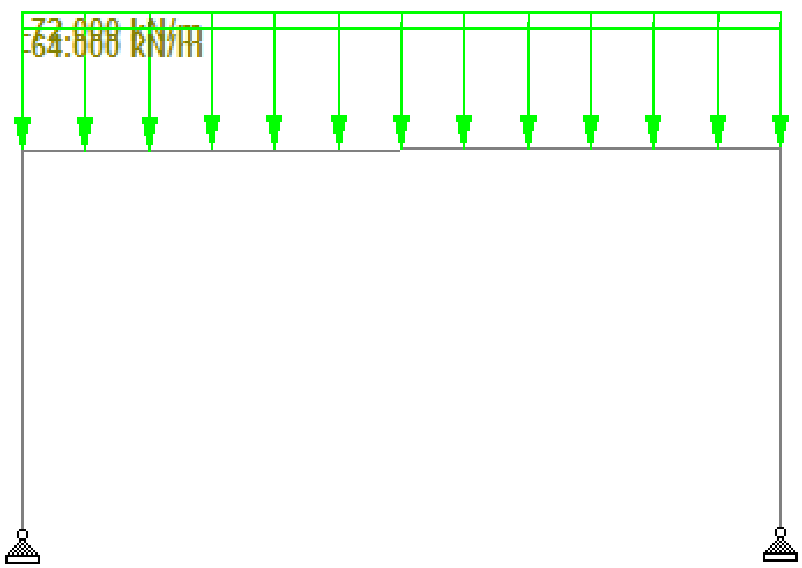
Use $f_c' = 28\ Mpa$ and $f_y = 420\ Mpa$.

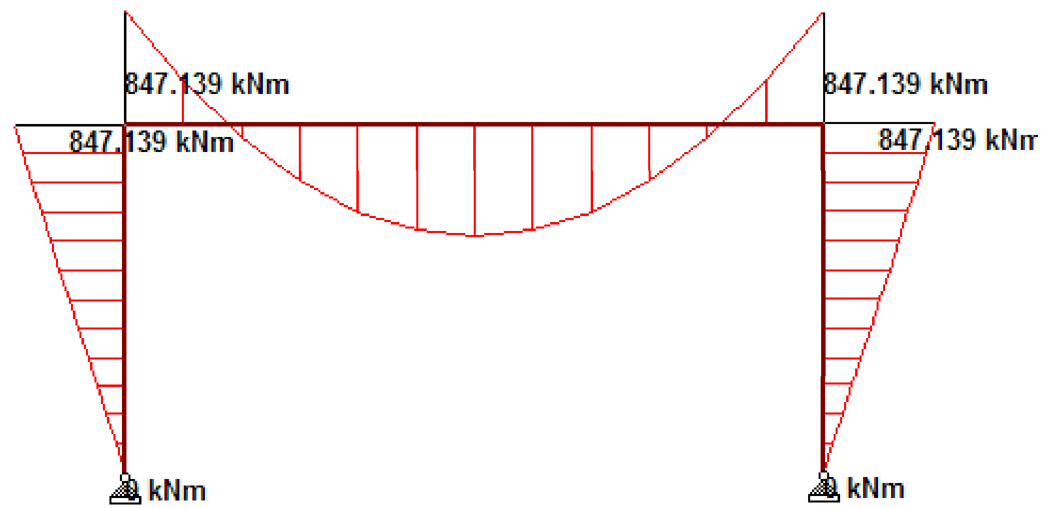
Note that all frame members are 30 x 60 cm in cross section.



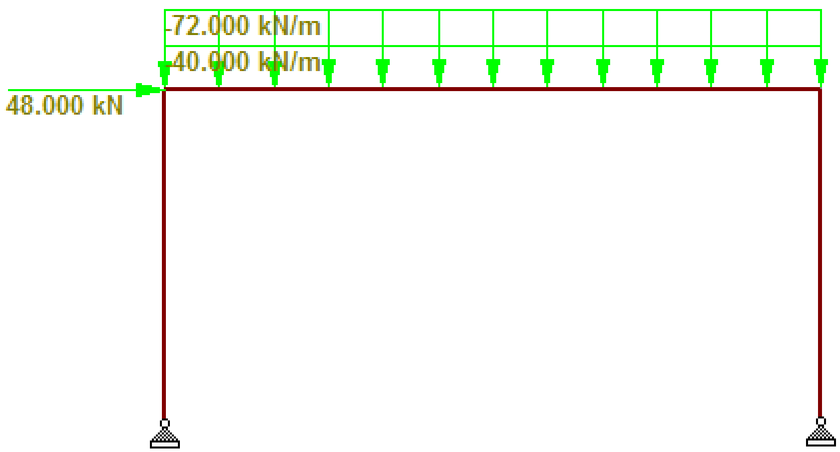
1- Using STAADpro-2004 structural software, the normal forces and bending moments for service dead, load, and wind loads are studied

1.2 D +1.6 L

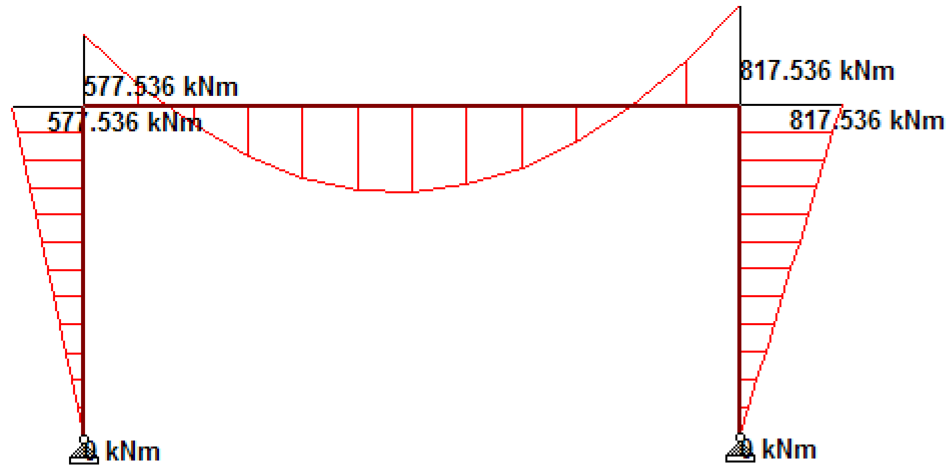




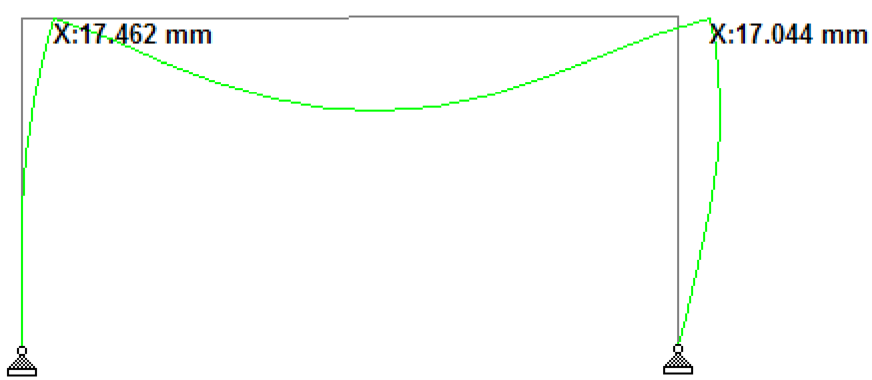
1.2 D +1.0 L+1.6W



-72.000 kN/m
-40.000 kN/m
48.000 kN

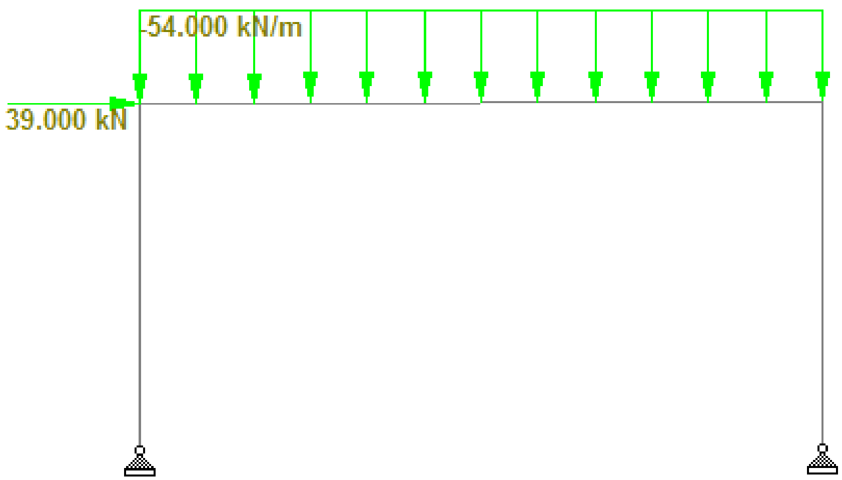


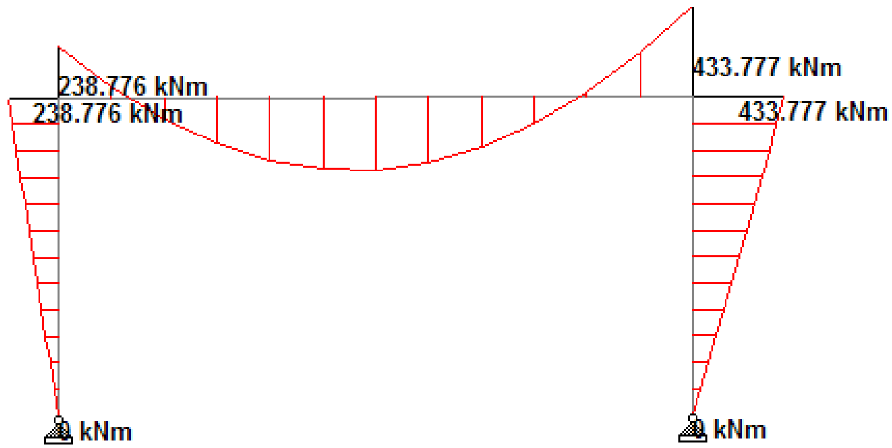
577.536 kNm
577.536 kNm
817.536 kNm
817.536 kNm
0 kNm
0 kNm

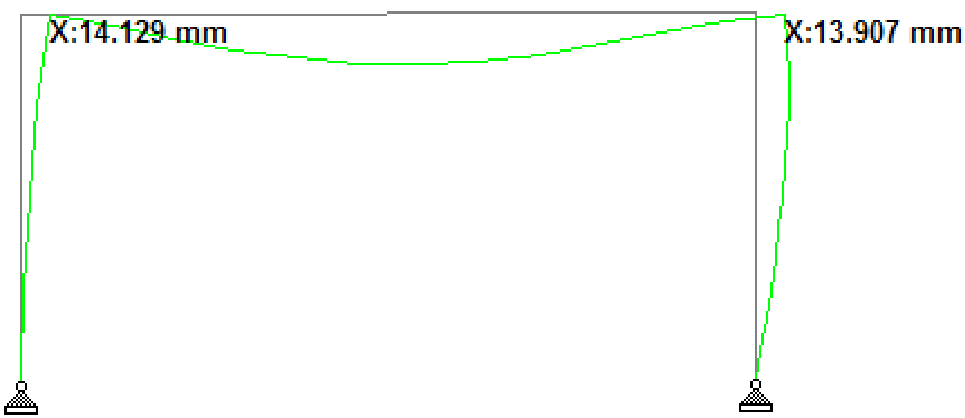


X:17.462 mm
X:17.044 mm

0.9 D +1.3 W


54.000 kN/m
39.000 kN


238.776 kNm
238.776 kNm
433.777 kNm
433.777 kNm
0 kNm
0 kNm


X:14.129 mm
X:13.907 mm

$case \rightarrow 1$

$1.2D + 1.6L$

$w_u = 136KN$

$M_u = 847KN.m$

$\Delta_x = 0$

$case \rightarrow 2$

$1.2D + 1.0L + 1.6W$

$w_u = 112KN$

$M_u = 818KN.m$

$\Delta_x = 17.5mm$

$case \rightarrow 3$

$0.9D + 1.3W$

$w_u = 54KN$

$M_u = 434KN.m$

$\Delta_x = 14.1mm$

2- Check whether columns on the second floor are sway or nonsway:

Case (2): 1.2D + 1.0L + 1.6W

For this case, the drift at corner = 17.5mm, evaluated using STAADpro2004.

The stability index $Q = \dfrac{\sum P_u \Delta_o}{V_u l_c}$

$$Q = \frac{\{112 \times 10\}(17.5 \times 10^{-3})}{\{1.6 \times 30\}(5.0)} = 0.082 > 0.05$$

Case (3): 0.9D + 1.3W

For this case, the drift at corner = 14.1mm, evaluated using STAADpro2004The

$$Q = \frac{\{54 \times 10\}(14.1 \times 10^{-3})}{\{1.3 \times 30\}(5.0)} = 0.04 < 0.05$$

i.e., the story is unbraced (sway) in case 2 and nonsway in case 1 & 3.

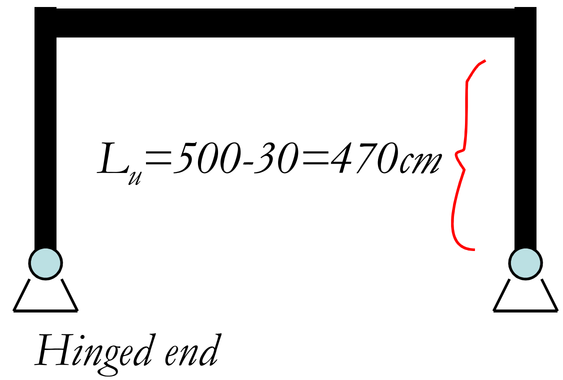
3- Check whether column is short or long:

Case 2 is (Sway case)

$$\psi = \frac{\left(0.7(30)(60)^3/12(500)\right)}{\left(0.35(30)(60)^3/12(1000)\right)} = 4.0$$

$$\boxed{k = 2.0 + 0.3(4) = 3.2}$$

$$\frac{k\, l_u}{r} = \frac{3.2(4700)}{0.3(600)} = 83.55 \quad > 22 \quad \text{Long column}$$



Case 1 & 3 is (nonsway case)

$$\boxed{k = 1}$$

$$\frac{k\, l_u}{r} = \frac{1(4700)}{0.3(600)} = 26.1$$

$$34 - 12\left(\frac{M_1}{M_2}\right) = 34 - 12\left(\frac{0}{M_2}\right) = 34$$

Short Column

Sway and nonsway Loads & Moments:

Case (1): 1.2 D +1.6 L

$$M_{ns} = 847 \ kN.m$$

$$M_s = 0$$

$$P_u = 680 \ kN$$

Case (2): 1.2 D + 1.0 L +1.6 W

$$M_{ns} = 698 \ kN.m$$

$$M_s = 120 kN.m$$

$$P_u = 560 \ kN$$

Case (3) : 0.9 D + 1.3 W

$$M_{ns} = 336 kN.m$$

$$M_s = 98 kN$$

$$P_u = 270 kN$$

4- Evaluate the magnified moments:

Case 2 need moment modification

$$\beta_d = \frac{1.2(60)}{1.2(60)+1.6(40)} = 0.52$$

$$E_c = 4775\sqrt{28} = 25267\ Mpa$$

$$EI = \frac{0.4(25267)(300)(600)^3}{12(1+0.52)} = 3.6(10)^{13}\ N.mm^2$$

$$P_{cr} = \frac{\pi^2(3.6)(10)^{13}}{(3.2\times 4700)^2(1000)} = 1570.7kN$$

$$\delta_s = \frac{1}{1-\dfrac{2*560}{0.75(2*1570.7)}} = 1.91$$

Modified Loads & Moments:

Case (1): 1.2 D +1.6 L

$$M_{\max} = 847\ kN.m$$

$$P_u = 680\ kN$$

Case (2): 1.2 D + 1.0 L +1.6 W

$$M_{\max} = M_{ns} + \delta_s M_s$$

$$698 + (1.91)120 = 927.2 kN.m$$

$$P_u = 560\ kN$$

Case (3) : 0.9 D + 1.3 W

$$M_{\max} = 434 kN.m$$

$$P_u = 270 kN$$

5- Design the reinforcement:

$$\gamma = \frac{60 - 2(4) - 2(1) - 2.0}{60} = 0.80$$

Case 1

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{560(1000)}{300(600)} = 3.11 Mpa = 0.44 ksi$$

$$\frac{\phi M_n}{A_g\, h} = \frac{M_u}{A_g\, h} = \frac{927.2(10)^6}{300(600)(600)} = 8.6 Mpa = 1.22 ksi$$

$$\rho = 0.07$$

$$A_S = 0.07 * 300 * 600 = 12600 mm^2 = 126 cm^2$$

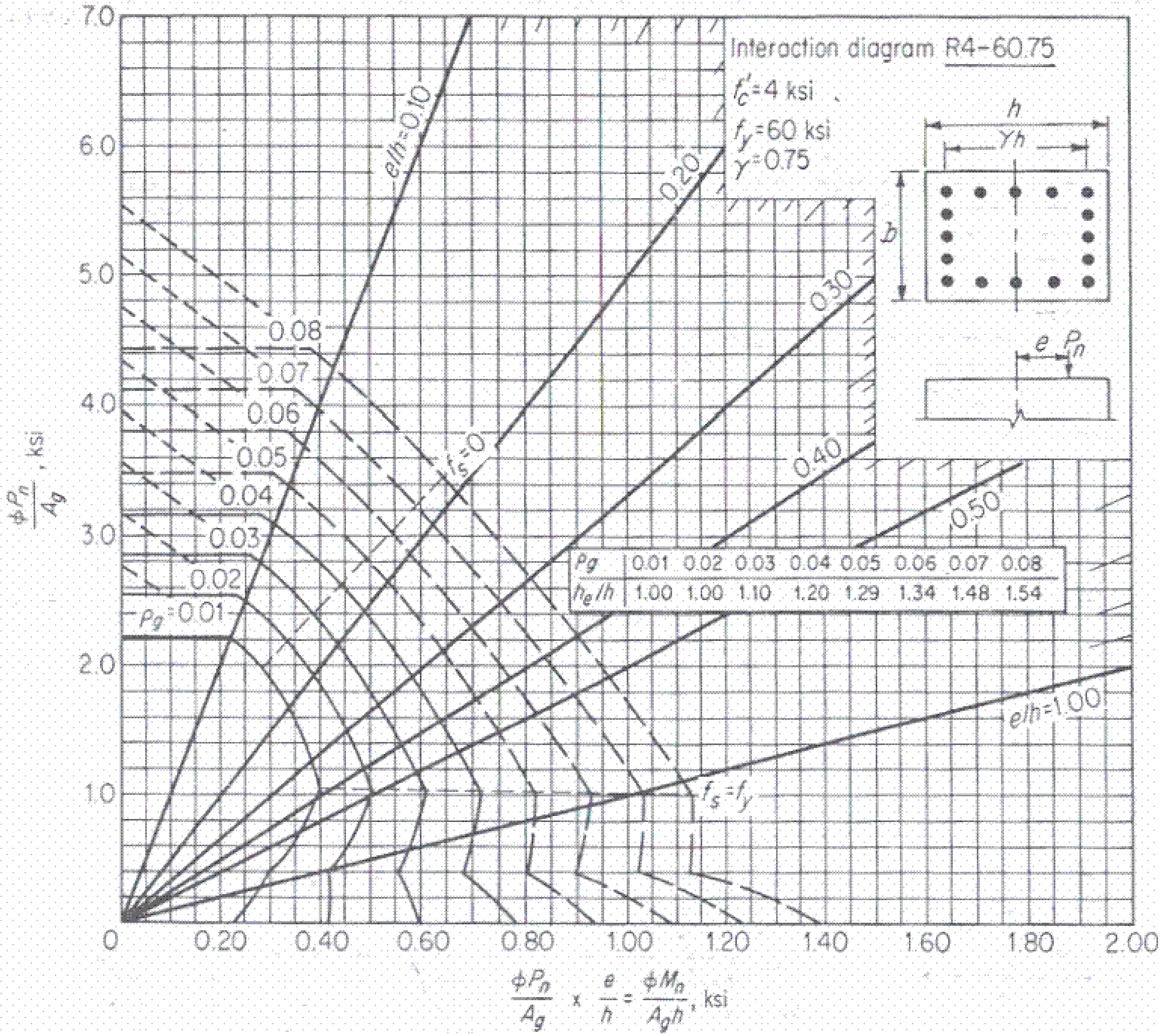
Case 2

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{680(1000)}{300(600)} = 3.78 Mpa = 0.54 ksi$$

$$\frac{\phi M_n}{A_g\, h} = \frac{M_u}{A_g\, h} = \frac{847(10)^6}{300(600)(600)} = 7.84 Mpa = 1.12 ksi$$

$\rho > 0.08$

Re*design*



Interaction diagram R4-60.75
f'_c=4 ksi
f_y=60 ksi
γ=0.75
h
γh
b
e P_n
e/h=0.10
0.20
0.30
0.40
0.50
e/h=1.00
f_s=0
f_s=f_y
0.08
0.07
0.06
0.05
0.04
0.03
0.02
ρ_g=0.01
ρ_g 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08
h_e/h 1.00 1.00 1.10 1.20 1.29 1.34 1.48 1.54
$\frac{\phi P_n}{A_g}$, ksi
$\frac{\phi P_n}{A_g} \times \frac{e}{h} = \frac{\phi M_n}{A_g h}$, ksi
0 0.20 0.40 0.60 0.80 1.00 1.20 1.40 1.60 1.80 2.00
1.0 2.0 3.0 4.0 5.0 6.0 7.0